Appendix B
Spherical coordinates

Abstract

This appendix presents a mathematical abstract, about spherical coordinates. This is necessary for a better understanding of the computations performed in section 3.3.2 about the input power.

B.1. Introduction of spherical coordinates

The spherical coordinates reference system is used in situation when important things about a point are its distance from the origin and, using terms from geography, its latitude and longitude.

This reference system is designated as (rho, phi, theta) or (ρ, ϕ, θ), where each element has the following meaning:

- ρ – denotes the the point's distance from the origin.
- ϕ – is the angle of ascension, between the modulus of the point’s vector distance from the origin and the *z*-axis. ϕ is positive for positive values of *z*-axis, and f ∈ [0, π].
- θ – is the angle of declination, or the angle from *xz* plane to the point.

Figure B.1 shows a graphical approach of the definition of the spherical coordinates.

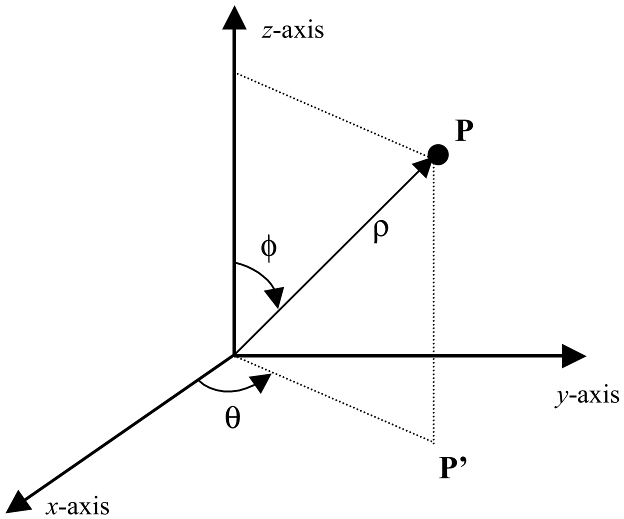


Figure B.1 Definition of spherical coordinates

B.2. Conversion of coordinate systems

Figure B.2 shows the conversion from spherical to cartesian coordinates and vice-versa. Figure B.2b, on the right shows the *xy*-plane from the figure B.2a, on the left.

Notice that, by the Pythagorean Theorem:

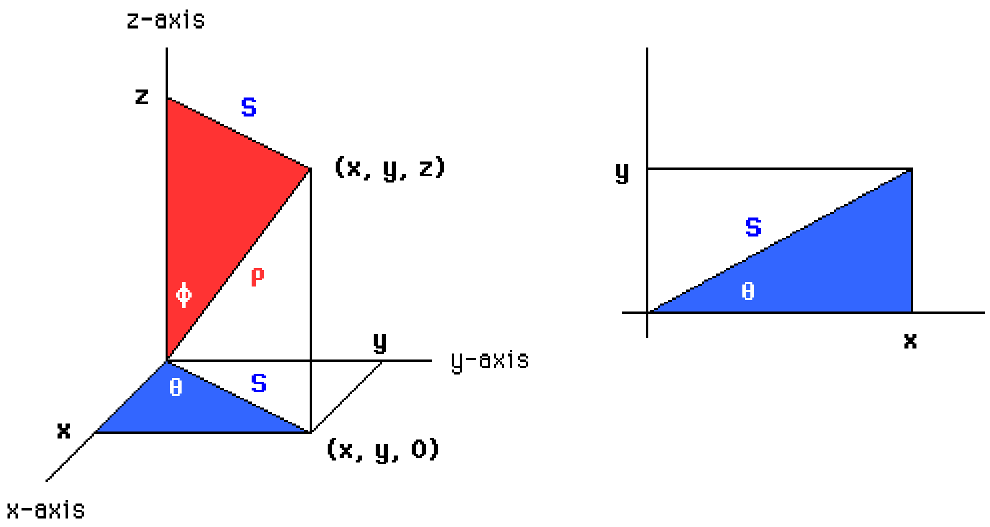
$$S = \sqrt{x^2 + y^2} = \rho \cdot \sin\phi \tag{B.1}$$

$$\rho = \sqrt{S^2 + z^2} = \sqrt{x^2 + y^2 + z^2} \tag{B.2}$$

$$x = S \cdot \cos\theta = \rho \cdot \sin\phi \cdot \cos\theta \tag{B.3}$$

$$y = S \cdot \sin\theta = \rho \cdot \sin\phi \cdot \sin\theta \tag{B.4}$$

$$z = \rho \cdot \cos\theta \tag{B.5}$$



a b

Figure B.2 Transformation of coordinate reference system
[http://www.math.montana.edu/]

The last three equations, (B.3)÷(B.5) represents the conversion from spherical to cartesian coordinates.

The conversion from cartesian to spherical coordinates is achieved through the following equations:

$$\rho = \sqrt{x^2 + y^2 + z^2} \tag{B.6}$$

$$S = \sqrt{x^2 + y^2} \tag{B.7}$$

$$\phi = \arccos\left(\frac{z}{\rho}\right) \tag{B.8}$$

$$\theta = \begin{cases} \arcsin\left(\frac{y}{S}\right), & \text{if } 0 \le x \\ \pi - \arcsin\left(\frac{y}{S}\right), & \text{if } x < 0 \end{cases} \tag{B.9}$$